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Influence of terrain on scaling laws for river networks

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[1] The upper Cheat River network departs from scaling laws describing a large number of river networks in North America. This departure is traced to its corrugated terrain. The more typical random terrain of the lower Cheat River network obeys the standard scaling laws. We modify the random network model of Scheidegger to include the effects of topography, reproducing the behavior observed in the Cheat River basin. *INDEX TERMS:* 1824 Hydrology: Geomorphology (1625); 1848 Hydrology: Networks; 3299 Mathematical Geophysics: General or miscellaneous; *KEYWORDS:* river networks, river basins, self-organization, power laws

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1. Introduction

[2] Scaling laws are important tools for understanding the complexity of river networks [*Dodds and Rothman*, 1999; *Rodriguez-Iturbe and Rinaldo*, 1997]. The availability of digital elevation models has provided ways to easily obtain the structure of large river networks [*Rodriguez-Iturbe et al.*, 1992; *Tarboton et al.*, 1991]; therefore it is useful to obtain simple relations to understand their features. Large numbers of data have to be organized and classified for the exploration and comparison of different river networks. To this end, scaling relations have proved to be unifying concepts. Several scaling relations have been proposed, most of which are in the form of ratios or power laws. In this work we apply two of the most commonly used scaling laws, previously considered to be universal, to the Cheat River basin in northern West Virginia, and identify departures from scaling due to the corrugated terrain.

[3] A scaling relation based on ratios was introduced by *Horton* [1945]; this relation was later modified by *Strahler* [1952], and it is known today as the Horton-Strahler law. This relation is obtained by ordering the rivers as they flow from their sources. Rivers that originate in a source are assigned an order equal to one. As two rivers of order one join they form a river of order two. In general, every time two rivers of the same order join they form a river of the next higher order. When two rivers of different order join, they take the higher order. The Horton-Strahler law found that the ratio between the number of rivers of a given order (N_w) to the number of rivers of one order higher (N_{w+1}) is a constant for a given basin

$$\frac{N_w}{N_{w+1}} = R_N$$

For river networks is North America, the Horton-Strahler ratio varies from 3 to 5 [*Rodriguez-Iturbe and Rinaldo*, 1997].

[4] A second scaling relation is based on the drainage area for a certain point in the river network [*Rodriguez-Iturbe and Rinaldo*, 1997]. This relation is particularly suited for analysis of data from digital elevation models. As a river network is extracted from the digital terrain, every pixel is assigned a drainage direction to a neighboring pixel. Thus every pixel in the terrain is connected to another. The cumulative drainage area is obtained by counting the number of pixels draining to a particular location. The distribution of drainage areas was found to follow a power law; thus the cumulative distribution is of the form

$$P[A > a] \propto a^{-\beta}.$$

From an analysis of river basins, the coefficient β was found to be equal to 0.43 ± 0.02 [*Rodriguez-Iturbe and Rinaldo*, 1997].

[5] In this paper we will show how mountainous terrain in the Cheat River basin influences and modifies both the power law scaling of drainage areas and Horton's law. We used a random network model, called the Scheidegger model to be described below, to model the folded terrain.

2. The Cheat River Basin

[6] The Cheat river basin in West Virginia consists of a 4600 square-kilometer network draining into Cheat Lake. The structure of the network was obtained using a 1-Degree digital elevation model (DEM) from the United States Geological Survey. The DEM gives the average elevation of a 3 arc second by 3 arc second region, which corresponds to an average area of approximately 6570 square meters at these latitudes. We did not take into account the small surface area variation due to the differences in latitude throughout the Cheat River basin. The river network was extracted using the set of subroutines TARDEM developed by *Tarboton* [1997]. These routines assign a drainage direction to every pixel in the DEM. The TARDEM routines eliminate pits in digital elevation data using a "flooding" approach, the points inside the pit are raised to the lowest

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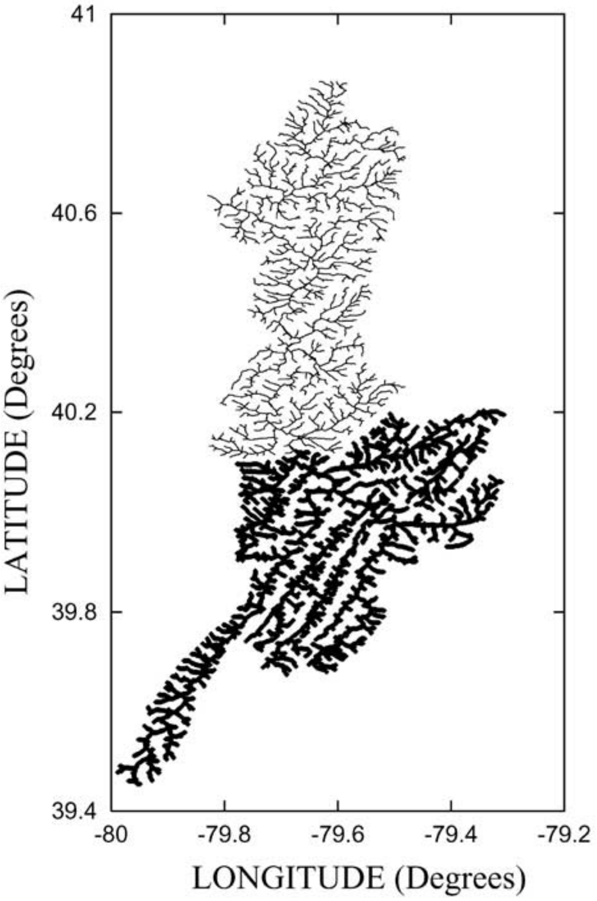

Figure 1. Cheat River network. The southern subnetwork is displayed with thick lines.

pour point in the perimeter. The total drainage area can be calculated by adding the total number of pixels that drain into a particular location. The collection of pixels with drainage area larger than a certain threshold form the river network. In Figure 1 we show the Cheat River network as extracted from the DEM. To study the effects of varying topography, we separated the network into two parts by choosing a subbasin located at the southern end of the Cheat River basin, consisting of the upper reaches of the basin. The separation was chosen so that the combined lengths of the rivers are the same in the southern and northern portions of the basin. The cumulative drainage area distribution for the entire network is shown in the upper trace of Figure 2. Figure 2 displays the number of pixels with drainage area larger than a certain threshold. The total distribution can be approximated by a power law $F[N > a] \propto a^{-\beta}$ with exponent $\beta = 0.40$, which fits very well for small drainage areas. However a departure from the power law lies near a drainage area of 4000 pixels. This departure is shown better by separate analyses of the northern and the southern portions of the network. Figure 2 also displays the cumulative distribution for the southern and the northern parts of the basin. The northern part clearly shows a power law behavior with exponent $\beta = 0.44$, while the southern part obeys scaling only over much smaller areas. The latter behavior is caused by the narrow valleys observed in the southern part, which precl e formation of large, quasi-isotropic drainage areas. In contrast, the northern portion of the network runs in terrain allowing the formation of larger drainage areas that are approximately isotropic.

[7] We computed Horton's stream order for every stream in the Cheat River basin. Our results are summarized in Figure 3, which indicate that Horton's law is obeyed. The lines on the Figure 3 indicate a least squares fit to the data; Horton's ratio corresponds to the inverse of the slope of the linear fit. We obtained a Horton's ratio equal to 4.9 for the entire Cheat River watershed (top line) which is consistent with the findings of several watersheds in North America [*Rodriguez-Iturbe and Rinaldo*, 1997]. To analyze the impact of terrain on Horton's ratio, we detached the southern portion of the network from the complete network (bottom line). The southern portion contains narrow valleys that do not allow the formation of large quasi-isotropic drainage areas. The streams in the southern subnetwork also obey Horton's law with Horton's ratio equal to 5.6. This indicates that Horton's law is still valid, although Horton's ratio is slightly higher than other river networks. The larger number for Horton's ratio in the mountainous region indicates that streams of lower order are more abundant as compared to the whole watershed. The ridges reduce the number of confluences of low-order streams to form higher-order streams.

3. Theoretical Models

[8] We studied the effects of the narrow valleys on models of river networks. To this end we used the Schei-

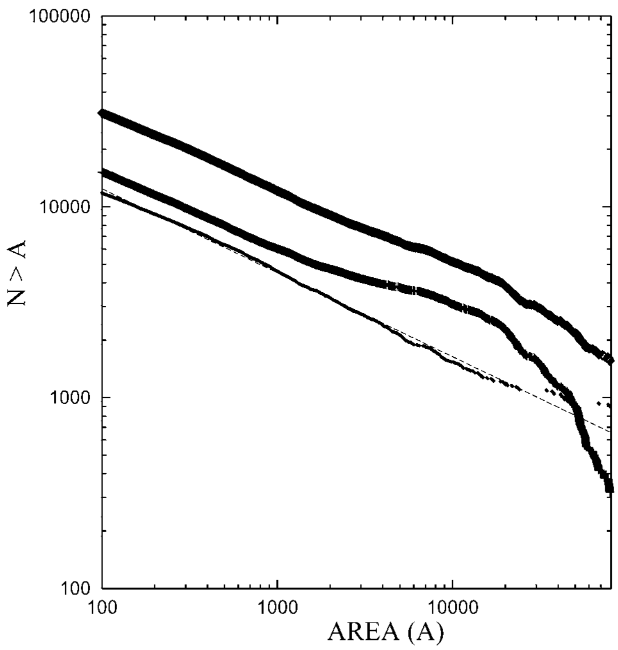

Figure 2. The cumulative distribution for drainage areas for the Cheat River watershed. We display the total number of pixels having a drainage area greater than the area on the horizontal axis. The unit area is one pixel of a digital elevation model corresponding to approximately an area of 6570 m^2. The line of top is the total network, the second line from the top corresponds to the southern portion of the network, and the third line is the northern portion of the network. The dashed line is a least squares fit.

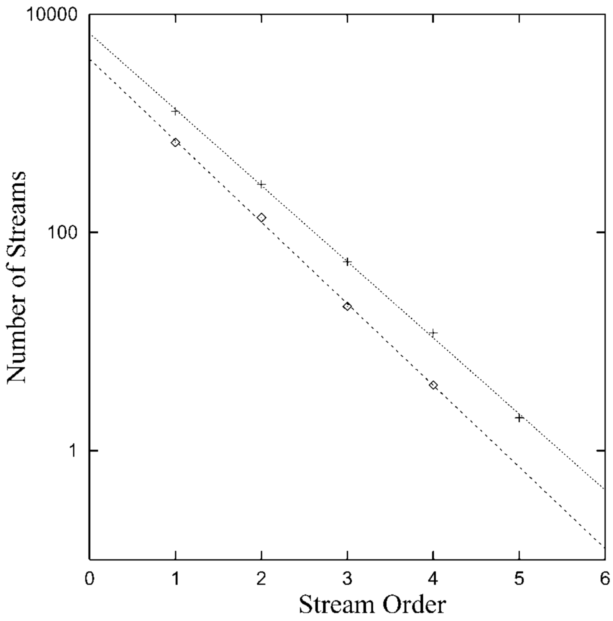

Figure 3. Horton's law for the Cheat River watershed. The plus signs indicate Horton's order for the entire watershed. The diamonds indicate the results for the southern subnetwork. The lines are least squares fits to each case.

degger model, which has been extensively studied in the literature [*Scheidegger*, 1967, 1991]. The Scheidegger model consists of a two-dimensional triangular lattice over which rivers flow from one edge, called the top, to the other edge called the bottom. The network is constructed by starting at the top edge and for each site on the top edge randomly choosing a direction of flow, either right or left. After constructing the first row, the next row is constructed in the same manner, and so on to construct the entire network. Previous studies of this model have shown that it obeys the standard geostatistics of river networks, although the corresponding exponents in the power laws are not the ones observed in nature [*Takayasu et al.*, 1988; *Nagatani*, 1993]. For example, the exponent β is equal to 1/3, instead of 0.43, as observed in typical river networks in North America [*Rodriguez-Iturbe and Rinaldo*, 1997]. We modified the Scheidegger model by imposing parallel ridges on it. A ridge will correspond to a line that separates waters flowing in opposite directions. To obtain the required statistics we took a lattice size of 70 by 140 lattice points and imposed 4 ridges. The ridges ran vertically from top to bottom and covered only the top half of the network.

[9] Figure 4 shows a small portion of the network, showing only one of the four ridges. The results from the cumulative area distributions are shown in Figure 5. We observe the same effect as observed in the Cheat river basin. If we consider the total Scheidegger network including the ridged portion, the cumulative area distribution is close to a power law. However if we separate the upper part and the lower parts of the network, the lower part follows a power law over a wider range of areas, contrary to the behavior of

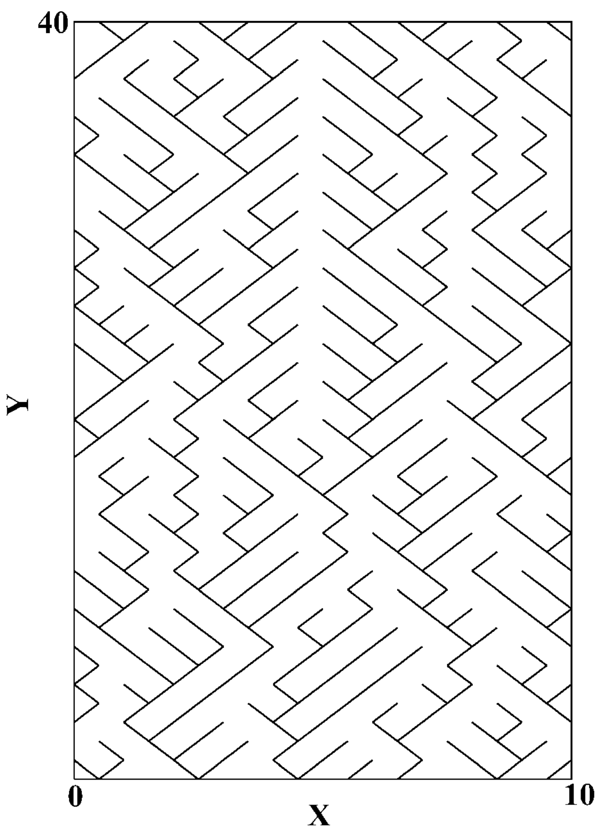

Figure 4. A Scheide etwork with a single ridge.

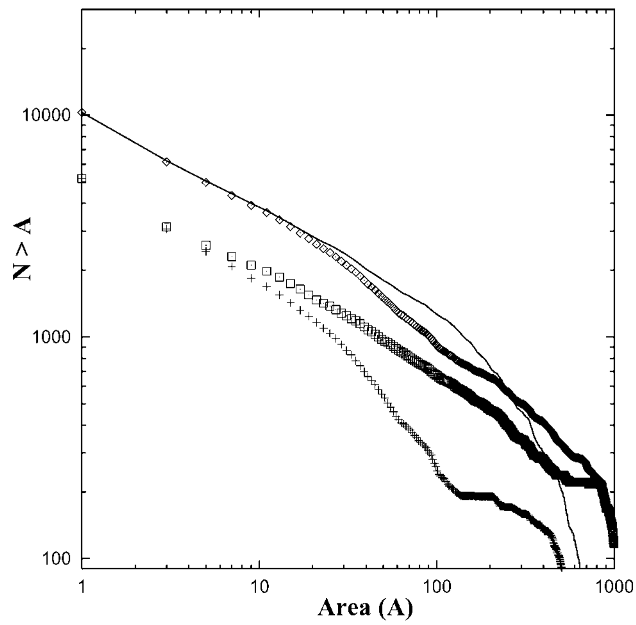

Figure 5. The cumulative distribution of drainage areas in a Scheidegger network. The solid line corresponds to the Scheidegger model. The diamonds are the distribution for the Scheidegger model with ridges. The squares include only the areas on noncorrugated terrain; the crosses correspond to areas on narrow valleys.

the upper part, which contains the ridges. The area with ridges deviates from the power law much more rapidly. We also compare the results for a lattice without ridges, showing very smooth behavior for the cumulative distribution of areas.

[10] To show that the results of Figure 5 are clearly the product of narrow valleys in the Cheat River basin, we used the basin boundaries as the boundaries for a random network model. We imposed four narrow valleys on the southern portion of the basin, as shown in Figure 6. The imposed valleys are parallel to each other, their direction does not follow the direction of the actual terrain. This random model uses a square lattice, instead of the triangular lattice used in the Scheidegger model. In the square-lattice model every point of the lattice is connected randomly to one of its eight nearest neighbors. Then the network is checked to determine whether it contains any loops. If it contains one or more loops, the directions are reassigned until a loopless lattice is formed. The network outlet was chosen near the geographical outlet of the Cheat River watershed.

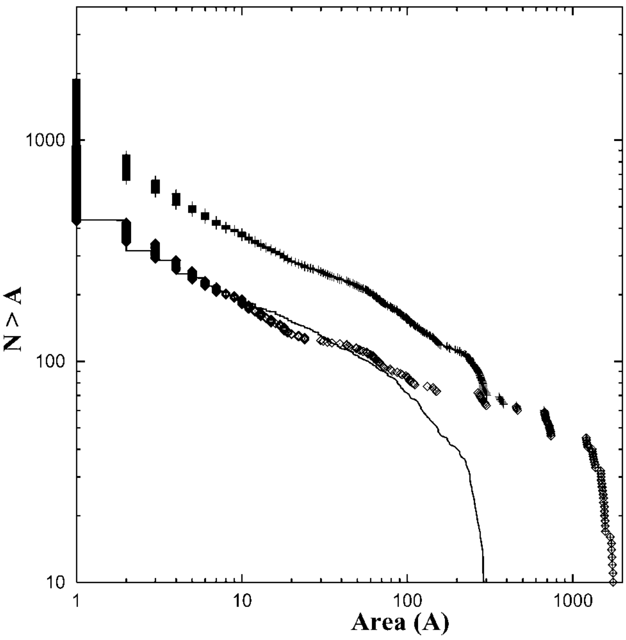

Figure 7. The cumulative distribution of drainage areas in the random network with the Cheat basin boundaries. The data on top (plus signs) correspond to the southern region with imposed ridges. The data below (diamonds) are for the portion without ridges (northern portions)). The solid line corresponds to the portion with ridges (southern portion). The unit area corresponds to one pixel of the model network.

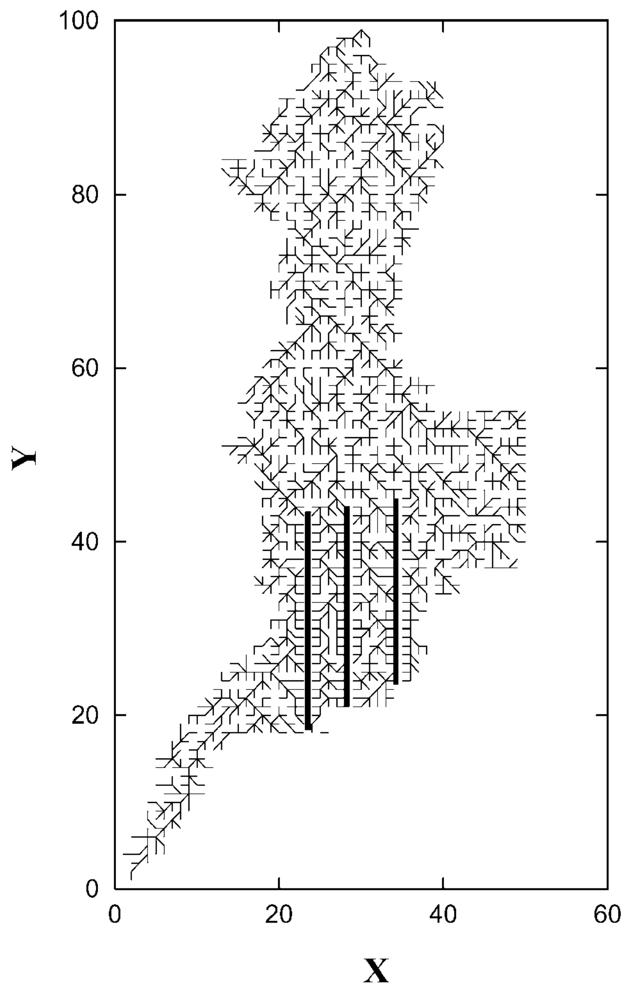

Figure 6. A random network using the boundaries of the Cheat River watershed. We imposed three ridges in the southern portion of the watershed. The unit area corresponds to one pixel of the l network.

[11] In Figure 7, we show the cumulative distribution of drainage areas for the random network. We compare the results for the total network, the portion with ridges and the portion without ridges. Our results show clear departures from scaling in the network with ridges. Therefore we conclude that the corrugation imposed on the model is what causes the departure from scaling. This behavior is similar to the behavior observed for the Cheat River watershed.

4. Conclusions

[12] We showed the relevance of the terrain to the scaling laws for river networks. Analyzing the northern portion of the Cheat River watershed in northern West Virginia, we found that the drainage area distribution follows a power law behavior, except in the southern portion of the watershed. This difference is attributed to the narrow parallel valleys in the southern part, which precludes the formation of larger quasi-isotropic drainage areas. We have verified that this behavior is observed in theoretical models of river networks in which narrow valleys and ridges are imposed on portions of the basin. Consequently, the large scale features of the geography are very important elements that contribute to the morphology of the river network.

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